Int. J. Heat Mass Transfer. Vol. 18, pp. 1277–1284. Pergamon Press 1975. Printed in Great Britain

APPLICATION OF A TRANSITIONAL BOUNDARY-LAYER THEORY IN THE LOW HYPERSONIC MACH NUMBER REGIME*

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(*Received* 13 *May* 1974 *and in revised form* 20 *February* 1975)

Abstract—An investigation is made to assess the capability of a finite-difference boundary-layer procedure to predict the mean profile development across a transition from laminar to turbulent flow in the low hypersonic Mach number regime. The boundary-layer procedure uses an integral form of the turbulence kinetic energy equation to govern the development of the Reynolds apparent shear stress. The present investigation shows the ability of this procedure to predict Stanton number, velocity profiles, and density profiles through the transition region and, in addition, to predict the effect of wall cooling and Mach number upon transition Reynolds number. The investigation also examines the contribution of the pressure dilatation term to the energy balance and suggests that transition can be initiated from the direct absorption of acoustic energy even if only a small amount (1 per cent) of the incident acoustic energy is absorbed.

NOMENCLATURE

a_n, structural coefficient of turbulence;
C_p, specific heat;
$\mathscr{D}_1$, sublayer damping factor;
$\mathscr{D}_2$, low Reynolds number correction factor;
k, thermal conductivity;
L, dissipation length;
l, mixing length;
M, Mach number;
M_r, Mach number relative to free stream;
Pr, Prandtl number;
Pr_t, turbulent Prandtl number;
P, pressure;
Q, heat flux;
q^2, turbulence kinetic energy;
Re_x, Reynolds number based upon streamwise distance;
R, turbulence Reynolds number;
St, Stanton number;
T, static temperature;
T^0, total temperature;
u, streamwise velocity;
v, transverse velocity;
w, cross flow velocity;
y, transverse coordinate;
y^+, dimensionless transverse coordinate.

Greek symbols

γ, ratio of specific heat;
δ, boundary-layer thickness;
δ_s, sublayer thickness;
δ^+, reference length;
ε, turbulence dissipation;
η, dimensionless transverse coordinate, y/δ^+;
μ, viscosity;
ν, kinematic viscosity;
ν_t, kinematic eddy viscosity;
ρ, density;
τ, shear stress;
ϕ_1, ϕ_2, ϕ_3, integral functions.

Subscripts

e, edge condition;
∞, free stream condition.

INTRODUCTION

TRANSITIONAL boundary layers play an important role in the successful design and operation of hypersonic re-entry vehicles. During transition both the wall shear and wall heating can reach their peak values, thus having a potentially important effect upon vehicle drag and the amount of cooling required to maintain structural integrity. In addition, the wake structure behind the vehicle which depends upon the wall boundary-layer development may vary significantly with transition location. Thus an analytical procedure capable of predicting the development of high Mach number boundary layers through the laminar, turbulent, and *transitional* regime would be a very useful tool in vehicle design.

A number of numerical procedures exist to predict the development of laminar boundary layers and a variety of turbulence models based upon a large amount of experimental data have allowed these procedures to be extended to the prediction of turbulent flows (e.g. [1]). However, to date most prediction procedures which have considered transitional boundary layers have been highly empirical [2]. These procedures usually trigger transition at a specified momentum thickness (or displacement thickness) Reynolds number which may be a function of pressure gradient,

*This work was sponsored by NASA Langley Research Center under Contract No. NAS1-10865.
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free-stream turbulence level, etc. The length of the transition region and the shear stress within this region are also usually set empirically.

Although these empirical transition models can give reasonable predictions for flows which correspond fairly closely to the flows from which the correlating data were obtained, such empirical models obviously have very limited applicability. The need for a more general transitional boundary-layer calculation procedure has led to the development of a model based upon a solution of the turbulence kinetic energy equation by McDonald and Fish [3]. Although the McDonald–Fish model also contains empiricism, the required assumptions are based upon a reasonable physical model and are made on a basic level, thereby, increasing the probability that these assumptions are valid over a wide range of flow conditions. Other similar transitional boundary-layer approaches based upon a turbulence kinetic energy concept have been initiated by Glushko [4] and Donaldson [5], however, neither the work of [4] or [5] has developed into a practical prediction procedure. A series of predictions for subsonic and moderately supersonic boundary layers has been presented in [3]. In view of the success of these calculations it was of interest to see if the same model could be used in the low hypersonic Mach number regime and, thus, the present study was undertaken.

The existence of phenomena associated primarily with hypersonic flow makes a straightforward application of any existing low Mach number analysis uncertain in the hypersonic flow regime. For example, in subsonic flow the free-stream turbulence kinetic energy, a vorticity disturbance mode, plays an important role in determining transition location [6]. At high Mach numbers, in addition to the vorticity mode, disturbances take the form of sound and entropy modes. The effect of the entropy mode upon transition is expected to be very small except perhaps at very high Mach numbers; however, as shown by Wagner, Maddalon and Weinstein [7], even at high Mach numbers the entropy disturbance may remain small. However, even at moderately supersonic Mach numbers the acoustic mode may play an important role in determining transition location. The acoustic mode may also be important in subsonic flow when the vorticity disturbance mode is small. A second phenomenon at high Mach numbers is the effect of pressure fluctuations upon the turbulence kinetic energy balance. In the present investigation preliminary assessments are made of both the direct absorption of acoustic energy and the effect of pressure dilatation upon the turbulence energy balance. In addition, comparisons between theory and experiment for velocity profiles, density profiles, and Stanton number are presented and an assessment of the boundary-layer procedure's ability to predict the initiation of transition in the low hypersonic Mach number regime is made.

Finally, the literature on transition is replete with anomalous experimental studies which would dishearten anyone attempting to discern subtle cause–effect relationships. However, it must be observed that, in spite of the known sensitivity of transition to the mode of excitation, in study after study the levels of the disturbances which could precipitate transition are not known. Inevitably this has led to a transition mystique and the thought that transition may not be a deterministic phenomena. This transition mystique persists in spite of the clear evidence that transition is not a haphazard phenomena; for instance, the transition Reynolds numbers based on displacement thickness Reynolds number correlate extremely well from a large number of different sources when the principal disturbing mode is free-stream turbulence or wall roughness. Where the difficulty lies is in attempting to evaluate the effect of, say, wall cooling on transition from various sources where the vorticity, acoustic and entropy fluctuation levels are unknown. The present approach is, for the present, highly simplified, taking no account of the spectral composition of the disturbance and concerning itself only with the disturbance overall energy level. In spite of these simplifications, the overall results are encouraging and the treatment of transition probably no worse than, say, the use of Prandtl's mixing length to describe fully-developed turbulent shear flows.

THEORY

The present investigation solves the usual set of boundary-layer partial differential equations in conjunction with an integral turbulence kinetic energy equation to predict the development of flows through the laminar, transitional and turbulent regimes. Within the framework of boundary-layer theory, various authors have reduced the time-averaged Navier–Stokes equations to the compressible boundary-layer equations of motion. For two-dimensional or axisymmetric flows, steady in the mean, the boundary-layer approximations to the momentum, energy, and continuity equations become

$$\overline{\rho u}\frac{\partial \bar{u}}{\partial x}+\overline{\rho v}\frac{\partial \bar{u}}{\partial y} = -\frac{\mathrm{d}\bar{P}}{\mathrm{d}x}+\frac{\partial \tau}{\partial y} \tag{1}$$

$$\overline{\rho u}C_p\frac{\partial \bar{T}^0}{\partial x}+\overline{\rho v}C_p\frac{\partial \bar{T}^0}{\partial y} = \frac{\partial}{\partial y}(Q+\bar{u}\tau) \tag{2}$$

$$\frac{\partial \overline{\rho u}r^\alpha}{\partial x}+\frac{\partial \overline{\rho v}r^\alpha}{\partial y} = 0 \tag{3}$$

where x and y are streamwise and transverse coordinates, u and v are velocity components in the x and y directions, ρ is density, P is pressure, C_p is specific heat, T^0 is total temperature, r is radius of curvature, and the exponent α is zero for two-dimensional flows and unity for axisymmetric flows. The shear stress τ and the heat transfer, Q, are given by

$$\tau = \bar{\mu}\frac{\partial \bar{u}}{\partial y}-\bar{\rho}\overline{u'v'} = \rho(v+v_t)\frac{\partial \bar{u}}{\partial y} \tag{4}$$

$$Q = \bar{k}\frac{\partial \bar{T}}{\partial y}-\bar{\rho}C_p\overline{v'T'} = (k+k_t)\frac{\partial T}{\partial y} \tag{5}$$

where v and k are molecular kinematic viscosity, and molecular thermal conductivity, and v_t and k_t are the effective turbulent viscosity and thermal conductivity, respectively. When the flow is laminar, equations (1)–(5) are solved with ν_t and k_t equal to zero to determine the flow development. If the flow is transitional or turbulent, ν_t and k_t are set through specification of a turbulence model. In the present procedure ν_t and k_t are related through a turbulent Prandtl number

$$Pr_T = \bar{\rho} C_p \nu_t / k_t \tag{6}$$

and the eddy viscosity, ν_t, is calculated through use of the turbulence kinetic energy equation.

The turbulence kinetic energy equation is a conservation equation derived from the Navier–Stokes equations by writing the instantaneous quantities as a sum of mean and fluctuating parts. The ith Navier–Stokes momentum conservation equation ($i = 1, 2, 3$, referring to the three coordinate directions) is multiplied by the ith component of fluctuating velocity and the average of the resulting equations is taken. The three averaged equations are summed to obtain the turbulence kinetic energy equation. The derivation for boundary layer flows has been presented by Bradshaw and Ferris [8] and a derivation and discussion of the approximation for hypersonic boundary layers is given by Shamroth and McDonald [9]. When the turbulence kinetic energy equation is integrated across the boundary layer between the wall and the outer edge the result, as shown in [9], is

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\phi_1 \rho_e u_e^3 \delta^+}{2a_1}\right) = \rho_e u_e^3\left(\phi_2 - \phi_3 + \frac{E}{\rho_e u_e^3}\right) + \int_0^\delta \overline{P' \frac{\partial u_i'}{\partial x_i}}\,\mathrm{d}y \tag{7}$$

where

$$\phi_1 = \int_0^{\delta/\delta^+} \frac{\overline{\rho u}}{\rho_e u_e}\left(\frac{l}{\delta^+}\frac{\partial \bar{u}/u_e}{\partial \eta}\right)^2 \mathrm{d}\eta$$

$$\phi_2 = \int_0^{\delta/\delta^+} \frac{\bar{\rho}}{\rho_e}\left(\frac{l}{\delta^+}\right)^2\left(\frac{\partial \bar{u}/u_e}{\partial \eta}\right)^3\left(1 - \frac{l}{L}\right)\mathrm{d}\eta$$

$$\phi_3 = \int_0^{\delta/\delta^+} \frac{\bar{\rho}}{\rho_e}\left(\frac{a_2 - a_3}{a_1}\right)\left(\frac{l}{\delta^+}\frac{\partial \bar{u}/u_e}{\partial \eta}\right)^2 \frac{\delta}{u_e}\left(\frac{\partial \bar{u}}{\partial x}\right)_{y=\mathrm{cons}} \mathrm{d}\eta$$

$$E = \left[\tfrac{1}{2}\overline{q^2}\left(\overline{\rho u}\frac{\partial \delta}{\partial x} - \overline{\rho v}\right) - \overline{P'v'} - \tfrac{1}{2}\overline{(\rho v)' q^2}\right]_e. \tag{8}$$

The structural coefficients a_n, the dissipation length L, and the mixing length l, are defined by

$$-\overline{u'v'} = a_1 \overline{q^2}, \quad \overline{u'^2} = a_2 \overline{q^2}, \quad \overline{v'^2} = a_3 \overline{q^2}, \quad \overline{w'^2} = (1 - a_2 - a_3)\overline{q^2} \tag{9}$$

$$\varepsilon = (-\overline{u'v'})^{3/2}/L, \quad (-\overline{u'v'})^{1/2} = l\frac{\partial \bar{u}}{\partial y}.$$

For fully-developed turbulence the structural coefficients a_1, a_2 and a_3 are assumed constant having values 0·15, 0·50 and 0·20, respectively; for transitional flows a_1 becomes a variable.

The LHS of equation (7) represents the streamwise rate of change of turbulence kinetic energy and is derived from the advection term in the original partial differential equation. The term $\rho_e u_e^3 \phi_2$ represents the integral of shear stress turbulence production minus dissipation and $\rho_e u_e^3 \phi_3$ represents the normal stress production. The terms designated by E are the turbulent source terms resulting from disturbances imposed upon the boundary layer by the free stream. As shown in equation (8), E is the sum of two contributions, the first $(\overline{q^2}/2)(\overline{\rho u}\,\mathrm{d}\delta/\mathrm{d}x - \overline{\rho v})$ representing a free-stream velocity disturbance entrained by the boundary layer and the second $\overline{P'v'}$ representing direct absorption of acoustic energy. The term $\overline{(\rho v)' q^2}/2$, which represents entrainment of the velocity disturbance due to the fluctuating field, is expected to be negligible compared to mean flow entrainment, $\overline{q^2}/2(\overline{\rho u}\,\mathrm{d}\delta/\mathrm{d}x - \overline{\rho v})$. Calculations made with both source terms are presented subsequently.

The solution of the turbulence kinetic energy equation still requires specification of profiles for L and l and specification of the function a_1. The dissipation length, L, and mixing length, l, are given by

$$L = 0{\cdot}1\,\delta \mathscr{D}_1 \mathscr{D}_2 \tanh[ky/(0{\cdot}1\,\delta)], \quad l = l_\infty \mathscr{D}_1 \tanh[ky/l_\infty] \tag{10}$$

where k is the von-Karman constant, $\mathscr{D}_1$ is the sublayer damping factor given in terms of the normal probability integral function

$$\mathscr{D}_1 = \mathscr{P}^{1/2}\{(y^+ - \overline{y^+})/\sigma\}, \quad y^+ = \frac{y u_\tau}{\nu} \tag{11}$$

where $\overline{y^+}$ is taken as 23 and σ is taken as 8. $\mathscr{D}_2$ is a low Reynolds number correction factor given in terms of a turbulence Reynolds number, R_τ, by

$$\mathscr{D}_2 = 1{\cdot}0 + \exp[-1{\cdot}63 \ln f(R_\tau) + 9{\cdot}7]. \tag{12}$$

In equation (12), the function $f(R_\tau)$ is given by

$$f(R_\tau) = 68{\cdot}1 R_\tau + 614{\cdot}3 \quad R_\tau > 40$$
$$f(R_\tau) = 100 R_\tau^{0{\cdot}22} \quad R_\tau \leqslant 1. \tag{13}$$

For $1 < R_\tau \leqslant 40$, the two profiles are joined by a cubic constructed to match the function and slope at each end point. The turbulence Reynolds number, R_τ, is defined in terms of the boundary-layer thickness, δ, and the sublayer thickness, δ_s, by

$$R_\tau = \frac{1}{\delta}\int_0^\delta \nu_\tau \,\mathrm{d}y \Big/ \frac{1}{\delta_s}\int_0^{\delta_s} \nu \,\mathrm{d}y \tag{14}$$

where δ_s, the sublayer thickness, is taken as the location where the laminar stress has fallen to 4 per cent of the total stress.

The final quantity required to be specified is the structural coefficient a_1. As suggested in [3] and [9], a_1 is defined in terms of R_τ (see equation (13)) and a_0 by

$$a_1 = a_0[f(R_\tau)/100]/[1 + 6{\cdot}666 a_0 (f(R_\tau)/100 - 1)] \tag{15}$$

where a_0 is a function of wall-to-free-stream temperature ratio, as shown in Fig. 1. Further details of both the mean flow equations and the turbulence model are given in [3] and [9]. The functional form of a_0 is chosen so as to obtain good agreement between theory and the data of Zysina–Molozhen and Kuznetsova [10] for the effect of wall temperature upon transition location. A comparison between theoretical predictions and experimental data is presented in Fig. 2.

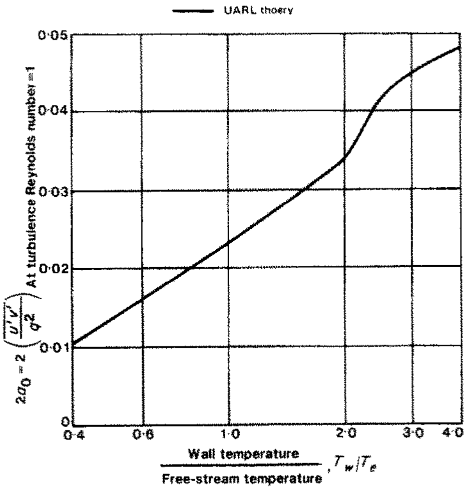

FIG. 1. Variation of structural coefficient with temperature ratio.

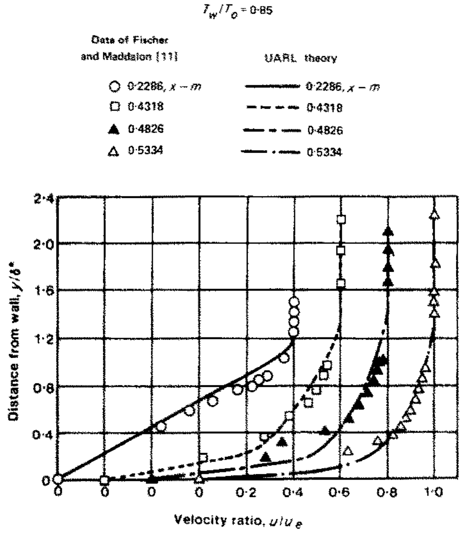

FIG. 3. Comparison between measured and predicted transitional velocity profiles at $M_e = 6·2$.

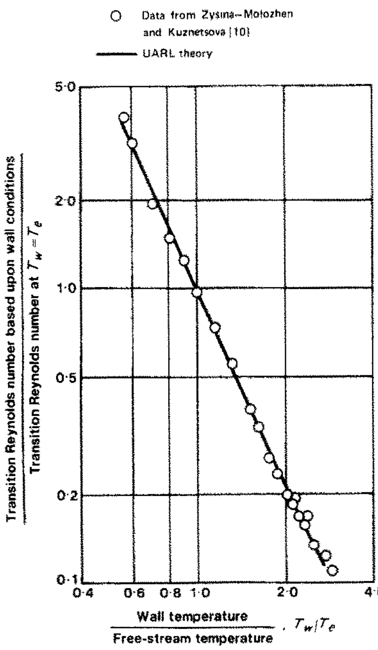

FIG. 2. Variation of transition Reynolds number with wall temperature ratio for incompressible flow.

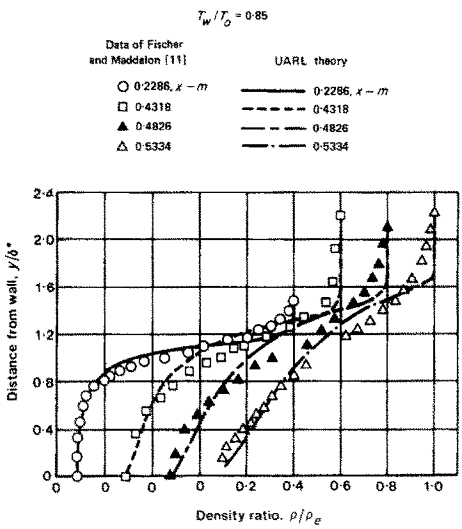

FIG. 4. Comparison between measured and predicted transitional density profiles at $M_e = 6·2$.

RESULTS

The procedure was assessed by comparing predicted velocity and density profiles and Stanton number distributions with experimental data for transitional boundary layers. A comparison between predictions of the theory and the measured velocity and density profiles of Fischer and Maddalon [11] for a Mach 6·2 transitional boundary layer is shown in Figs. 3 and 4. The calculation procedure requires an input disturbance to trigger transition. For the purpose of the comparison between the procedure and the data of Fischer and Maddalon [11], a disturbance level was set so as to trigger transition at the experimentally observed location. As can be seen in Fig. 3, the predicted length of transition and the predicted velocity profile are in very good agreement with the data (except near the wall where the measurements appear to be in error due to wall effects). Although the density profiles are not in as good agreement with data as the velocity profiles they are still quite acceptable, particularly considering the large density gradients through the mid portion of the boundary layer. A comparison between predicted Stanton number and the data of Stainback [12] for a Mach 5 boundary layer is shown in Fig. 5. In this calculation an input disturbance based upon the measured free-stream pressure fluctuation was used to trigger transition. The manner in which the pressure fluctuation is used to trigger transition is discussed subsequently. As shown in Fig. 5, the predicted length of the transition region is in good agreement with data. The predicted values of Stanton number appear to be

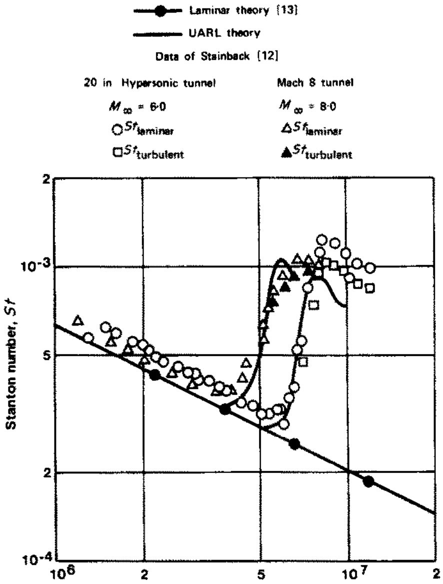

FIG. 5. Comparison between measured and predicted transitional heat transfer at $M_e = 5$.

approximately 15 per cent below the measured values in both the fully laminar and fully turbulent regime. However, in the fully laminar regime where one would expect very good comparisons with data the theoretical predictions are in excellent agreement with the analysis of Cohen [13] leading to the suspicion that the data may contain a systematic error. Finally, a comparison between the theoretical prediction and the Stanton number data of Holloway and Sterrett [14] is presented in Fig. 6 where the free-stream disturbance, as for the comparison presented in Figs. 3 and 4, was set so as to cause transition to occur at the experimentally observed

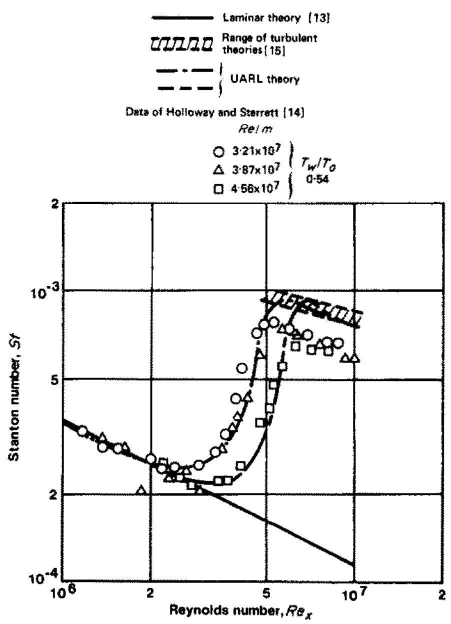

FIG. 6. Comparison between measured and predicted transitional heat transfer at $M_e = 4{\cdot}8$.

location. Once again both the predicted length of the transition regime and the predicted distribution of the Stanton number through transition are in good agreement with data; a discrepancy of about 15 per cent does exist between theory and experiment in the fully turbulent region. However, in the fully turbulent regime the predictions agree with the analysis of van Driest [15], whereas the data is lower.

As shown in Figs. 3–6, the theoretical predictions compare well with data through the transition regime; however, the additional question arises as to how well the analysis predicts initiation of transition. The results of McDonald and Fish [3] clearly show that the analysis predicts transitional location well for boundary layers in the low Mach number regime when the free-stream turbulence level, u'/u_e, is both the dominant disturbance and specified and, therefore, the theory has been developed for a free-stream velocity fluctuation input. Unfortunately, measurements of transitional boundary layers in the low hypersonic Mach number regime have not been accompanied by free-stream velocity fluctuation measurements and, in fact, the free-stream velocity disturbance may not even be the dominant disturbance. However, Stainback [16] has made simultaneous boundary-layer transition measurements and free-stream pressure fluctuation measurements to determine the effect of free-stream fluctuating pressure upon transition location. If the pressure disturbance propagates at a Mach number, M_r, relative to the free stream then the unsteady Bernoulli equation leads to a relation between fluctuating velocity and pressure in terms of the boundary-layer edge Mach number, M_e, and the relative Mach number, M_r (e.g. as shown by Laufer [17]) of the form

$$\left|\frac{u'}{u_e}\right| = \frac{K}{\gamma M_e M_r}\left|\frac{P'}{P_e}\right| \tag{16}$$

where, as discussed by Laufer [17] K^{-1} may be thought of as the integral of a space-time correlation function. The relative Mach number, M_r, and the tunnel Mach number, M_∞, are related by Laufer in [17]. If it is further assumed that the disturbance is an acoustic wave, v' and u' are related by

$$|v'| = |u'|\sqrt{(M_r^2-1)}. \tag{17}$$

Thus, when a plane wave disturbance is assumed and the tunnel Mach number, M_∞, and the boundary layer edge Mach number, M_e, are known, equations (16) and (17) along with

$$\overline{q^2} = \overline{u'^2}+\overline{v'^2}+\overline{w'^2} \approx \tfrac{10}{7}(\overline{u'^2}+\overline{v'^2}) \tag{18}$$

allow the measured pressure disturbance to be related to a velocity disturbance once a value of K is determined. The above procedure was used to relate the measured pressure fluctuation to an effective free stream velocity fluctuation required as input for the turbulence kinetic energy equation. This free-stream velocity fluctuation level was held constant throughout each run. Predictions of initiation of transition were then compared to the data of [16]; although [16] contained data from three different wind tunnels and two

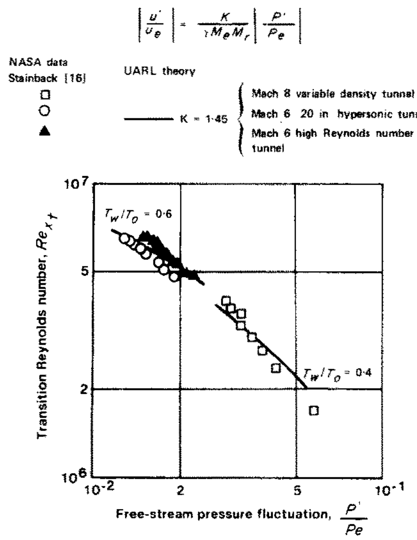

FIG. 7. Effect of free-stream fluctuating pressure on boundary layer transition location on sharp cones at $M_e = 5$.

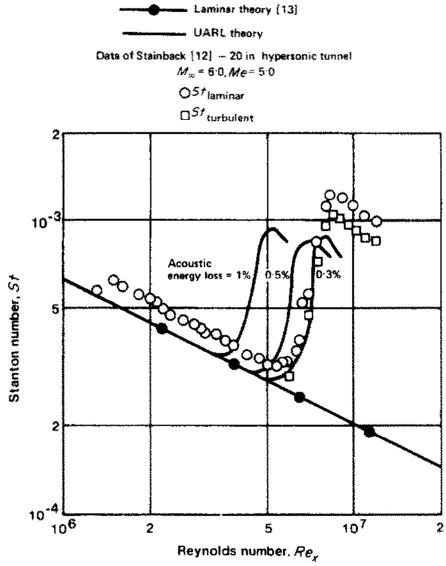

FIG. 8. Comparison between measured and predicted transitional heat transfer with transition triggered from pressure-velocity fluctuation.

different wall-to-free-stream temperature ratios, it was found that when K was taken to be 1·45, excellent agreement was obtained between the prediction and experiment, as shown in Fig. 7.

It should be noted that in the previous calculations of initiation of transition the entire disturbance was related to a velocity fluctuation disturbance and any direct absorption of acoustic energy was neglected. However, it is possible to trigger transition by assuming a percentage of the incident acoustic energy is absorbed by the boundary layer. In this model q^2 at the edge of the boundary layer is set equal to zero and $\overline{P'v'}$ in the source term, E (see equation (8)), is set equal to a nonzero value. For an incoming acoustic wave, $\overline{P'v'}$ is negative with respect to the boundary-layer coordinate system and its magnitude can be estimated from equations (16) and (17). The estimate was carried out assuming the factor K (see equation (16)) is unity and $\overline{P'v'} \simeq |P'||v'|$. Results for various amounts of absorbed acoustic energy are presented in Fig. 8 where theoretical predictions are compared with Stainback's data [12]. As can be seen, only a small amount of incident acoustic energy (for this calculation 0·3 per cent) need be absorbed to trigger transition at the experimentally determined streamwise station.

An assessment also was made of the effect of the pressure–dilatation term, $\overline{P'(\partial u_i'/\partial x_i)}$ upon transition [see equation (7)]. In incompressible flow this term is zero and at moderate Mach numbers it is expected to be negligible. In the previous calculations of this paper the pressure–dilatation term was neglected, however, since it may have a significant role in high Mach number flows its effect on transition location was estimated. As shown by Shamroth and McDonald [9], the pressure–dilatation term can be roughly approximated by

$$P'\frac{\overline{\partial u_i'}}{\partial x_i} = -\frac{\overline{P'v'}}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial y}. \tag{19}$$

The magnitude of $\overline{P'v'}$ is obtained under the assumption that the $\overline{P'v'}$ to be used must be that generated by the boundary layer rather than that propagating into the boundary layer from the free stream. For this purpose P' is estimated by

$$P' \approx \tau_w f(M_e)\frac{\int_0^\delta v_T\,\mathrm{d}y}{\int_0^\delta v_T + v\,\mathrm{d}y} \tag{20}$$

where τ_w is the wall shear stress and $f(M_e)$ is taken from the data of Kistler and Chen [18]. In addition, v' is estimated by

$$\overline{v'^2} = \frac{0\cdot 2}{\delta}\int_0^\delta \overline{q^2}\,\mathrm{d}y \tag{21}$$

and

$$\overline{P'v'} = |P'||v'|. \tag{22}$$

These approximations were used to include the effect of the pressure–dilatation term on transition; the results shown in Fig. 9 indicate that the effect is small.

Finally, a prediction of the effect of Mach number upon transition location at a given disturbance level and a given wall temperature ratio is compared with the data of Zysina–Molozhen and Kuznetsova [10] in Fig. 10. Although the procedure has not been adjusted for Mach number, the agreement between theory and experiment is very good.

It should be recalled that the analysis has been developed using the data of [10] to account for the effect of wall-to-free-stream temperature ratio in low Mach number flows. However, no specific adjustment of the analysis for the effect of Mach number was made. As shown in Fig. 10, the prediction of the analysis for the effect of Mach number upon transition location is in good agreement with the experimental measurements.

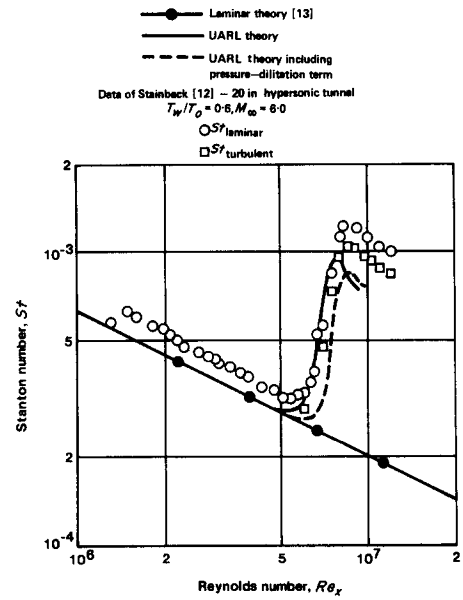

FIG. 9. Comparison between measured and predicted transitional heat transfer at $M_e = 5$ with and without pressure–dilitation term.

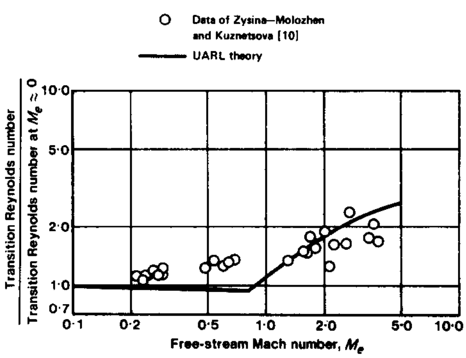

FIG. 10. Variation of transition Reynolds number with Mach number.

CONCLUSIONS

The comparisons between theoretical predictions and experimental data show that the present procedure can accurately predict velocity and density profiles as well as Stanton number distributions for low hypersonic Mach number boundary layers transiting from the laminar to the turbulent state. Despite the fact that the procedure does not consider the spectral composition of the free-stream disturbance and only considers the overall disturbance energy level, the procedure does show considerable promise for predicting transition Reynolds number when the free-stream disturbance level is specified. Finally, the procedure does give the correct effect of Mach number variation upon transition location and indicates that only a small percentage of available acoustic energy need be absorbed by the boundary layer to trigger transition directly. Obviously, further development and assessment of the theory, particularly in the supersonic and hypersonic regimes, must rely heavily upon experimental guidance. In particular, information of the type not generally measured in the transitional boundary layer, such as the development of the Reynolds stress and the precise makeup of disturbance modes is required. When such information is available the present procedure can be further assessed and developed.

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APPLICATION D'UNE THEORIE DE LA COUCHE LIMITE DE TRANSITION DANS LE REGIME DES FAIBLES NOMBRES DE MACH HYPERSONIQUES

Résumé—Une étude est effectuée afin d'évaluer la capacité d'une procédure de différences finies à prédire le développement des profils moyens dans la zone de transition laminaire–turbulent pour le régime de couche limite aux faibles nombres de Mach hypersoniques. La méthode de calcul de la couche limite utilise une forme intégrale de l'équation de l'énergie cinétique de turbulence afin de déterminer l'évolution de la tension de cisaillement de Reynolds. La présente étude montre que la procédure permet la prévision du nombre de Stanton, des profils de vitesse et des profils de densité à travers la zone de transition et, de plus, de prévoir les effets du refroidissement de paroi et du nombre de Mach sur le nombre de Reynolds de transition. L'étude examine également la contribution du terme de dilatation de pression dans le bilan d'énergie et suggère que la transition peut être provoquée par l'absorption directe d'énergie acoustique, même si une faible quantité (1 pour cent) de l'énergie acoustique incidente est absorbée.

DIE ANWENDUNG EINER ÜBERGANGS-GRENZSCHICHTTHEORIE IM BEREICH NIEDRIGER HYPERSONISCHER MACHZAHLEN

Zusammenfassung—Es wurde eine Untersuchung durchgeführt zur Abschätzung der Eignung einer Finite-Differenzen-Grenzschicht-Prozedur zur Vorhersage der Ausbildung des mittleren Profils beim Übergang von laminarer zu turbulenter Strömung im Bereich niedriger hypersonischer Machzahlen. Die Grenzschicht-Prozedur verwendet eine Integralform der Gleichung für die turbulente kinetische Energie zur Beschreibung der scheinbaren Schubspannung. Die vorliegende Untersuchung zeigt, daß diese Prozedur geeignet ist, Stanton–Zahlen, Geschwindigkeits- und Dichteprofile im Übergangsbereich vorauszuberechnen. Ferner ist es damit möglich, den Einfluß der Wandkühlung und der Machzahl auf die Übergangs-Reynolds–Zahl zu beschreiben. Die Untersuchung überprüft den Beitrag des Druck-Dilatations-Terms in der Energiebilanz. Es zeigt sich, daß der Umschlag selbst dann durch direkte Absorption akustischer Energie hervorgerufen wird, wenn nur ein kleiner Teil (1%) der einfallenden akustischen Energie absorbiert wird.

ИСПОЛЬЗОВАНИЕ ТЕОРИИ ПЕРЕХОДНОГО ПОГРАНИЧНОГО СЛОЯ В ОБЛАСТИ НИЗКОГО ГИПЕРЗВУКОВОГО ЧИСЛА МАХА

Аннотация — Анализируется возможность использования конечноразностного метода пограничного слоя для расчета среднего профиля скорости в области перехода от ламинарного течения к турбулентному при низких гиперзвуковых числах Маха. Метод пограничного слоя использует интегральную форму уравнения турбулентной кинетической энергии для определения напряжения сдвига Рейнольдса. Данное исследование подтверждает возможность использования этого метода для расчета числа Стантона, профилей скорости и плотности в переходной области, а также для расчета влияния охлаждения стенки в числа Маха на величину переходного числа Рейнольдса. Также исследуется влияние увеличения давления на баланс энергии и предполагается, что переход может возникнуть за счет непосредственного поглощения акустической энергии даже, если поглощается только небольшое количество (1%) падающей акустической энергии.